GMP small operands optimization

or

Is arithmetic assembly automatically optimal?

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GMP

What is GMP?

- A low-level bignum library for integers, rationals, and floats.
- Not an algebra system

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- A low-level bignum library for integers, rationals, and floats.
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GMP goals

- Correctness
- Performance
- Performance
- Performance

  ⋮

About this presentation

- Just about small operands
- Operands of 4097 bits are huge
- Quadratic algorithms

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- Just about small operands
- Operands of 4097 bits are huge
- Quadratic algorithms
- Karatsuba, who's that?

GMP primitives

Simple basic computation primitives:

- mpn_add_n
- mpn_sub_n
- mpn_mul_1
- mpn_addmul_1
- mpn_submul_1
- mpn_lshift
- mpn_rshift

GMP primitives

name	**operation**	**return value**
mpn_add_n	$A \leftarrow B + C$	carry-out
mpn_sub_n	$A \leftarrow B - C$	borrow-out
mpn_mul_1	$A \leftarrow B \times c$	most significant limb
mpn_addmul_1	$A \leftarrow A + B \times c$	most significant limb
mpn_submul_1	$A \leftarrow A - B \times c$	most significant limb
mpn_lshift	$A \leftarrow B \times 2^c$	“shifted out” limb
mpn_rshift	$A \leftarrow B/2^c$	“shifted out” limb

Use of primitives (example)

- Schoolbook multiplication made of:
  `mpn_mul_1`, $\{$`mpn_addmul_1`$\}*$
- Schoolbook left-to-right division made of:
  $\{\hat{q}$`-comp, mpn_submul_1, (mpn_add_n) or mpn_sub_n)`$\}*$
- Schoolbook right-to-left division made of:
  $\{q$`-comp, mpn_addmul_1`$\}*$`, mpn_add_n, (mpn_sub_n)`

GMP coding strategies

Use assembly code

GMP can use assembly for critical operations.

Use basic algorithms!

BAD:

```
if $n = 1$ then
    A[0] = B[0] * C[0]
else
    mul_sledgehammer (...)
```

GOOD:

```
if $n < 30$ then
    mpn_mul_basecase (...)
else
    mul_sledgehammer (...)
```

"All GMP code is rubbish"

All GMP code is due for replacement.

We're happy with improvements just for a day, or two.

Assembly in GMP

- C with inline assembly (GMP 1)
- $+$ simple assembly loops (GMP 2)
- $+$ complex assembly loops (GMP 3)
- $+$ nested loops (GMP 4)
- $+$ *OSP* loops (GMP 5)

Assembly strategies (1)

- Loop recurrency ”shallowing”
- Loop unrolling
- Software pipelining
- Loop unrolling + Software pipelining
- OSP(tm)

Recurrency shallowing — 3 operation mpn_add_n

```
add (word *r, word *u, word *v, size_t n)
{
  cy = 0;
  for (i = 0; i < n; i++)
    {
      uword = u[i];
      vword = v[i];
      sum0 = uword + cy;
      cy0 = sum0 < uword;
      sum1 = sum0 + vword;
      cy1 = sum1 < sum0;
      cy = cy0 + cy1;
      r[i] = sum1;
    }
}
```

Recurrency shallowing — 2 operation mpn_add_n

```
add (word *r, word *u, word *v, size_t n)
{
  cy = 0;
  for (i = 0; i < n; i++)
    {
      uword = u[i];
      vword = v[i];
      sum0 = uword + vword;
      cy0 = sum0 < uword;
      sum1 = sum0 + cy;
      cy1 = sum1 < sum0;
      cy = cy0 + cy1;
      r[i] = sum1;
    }
}
```

Recurrency shallowing — 1 operation mpn_add_n

```
add (word *r, word *u, word *v, size_t n)
{
  cy = 0;
  for (i = 0; i < n; i++)
    {
      uword = u[i];
      vword = v[i];
      sum0 = uword + vword;
      cy0 = sum0 < uword;
      sum1 = sum0 + cy;
      cy = cy0 | (cy & (sum0 == ~0));
      r[i] = sum1;
    }
}
```

Software pipelining of a tight loop

```
for (i = 1; i < n; i++)
  {
    a0 = ap[0];
    b0 = bp[0];
    r0 = a0 * b0;
    rp[i] = r0;
  }
```

Software pipelining of a tight loop

```
a0 = ap[0];
b0 = bp[0];

for (i = 1; i < n; i++)
  {
    r0 = a0 * b0;
    a0 = ap[i];
    b0 = bp[i];
    rp[i-1] = r0;
  }

r0 = a0 * b0;
rp[n - 1] = r0;
```

Loop unrolling + Software pipelining

```
   feed-in      |   pipelined loop           |  wind-down
----------------|----------------------------|-----------------
a0 = ap[0];     |   for (i = 4; i < n, i+=2) |
b0 = bp[0];     |     {                      |
a1 = ap[1];     |       rp[i-4] = r0;        |   rp[n-4] = r0;
b1 = bp[1];     |       r0 = a0 * b0;        |   r0 = a0 * b0;
                |       a0 = ap[i];          |   rp[n-3] = r1;
r0 = a0 * b0;   |       b0 = bp[i];          |   r1 = a1 * b1;
a0 = ap[2];     |       rp[i-3] = r1;        |
b0 = bp[2];     |       r1 = a1 * b1;        |   rp[n-2] = r0;
r1 = a1 * b1;   |       a1 = ap[i+1];        |   rp[n-1] = r1;
a1 = ap[3];     |       b1 = bp[i+1];        |
b1 = bp[3];     |     }                      |
```

Assembly strategies (2)

- Explore CPU pipelines
- (Read manufacturers' pipeline docs)
- Code for each pipeline
- Find "micro algorithms" for ISA/pipeline
- Work around pipeline flaws
- Avoid the branch misprediction death

Pentium 4 has a 10 cycle carry flag latency. . .

Unpredictable branches – BAD

```
if (carry_from_add)
    do something
else
    do some other thing
```

Unpredictable branches – OK

```
if (very_unlikely_condition)
   do something
else
   do some other thing
```

Assembly strategies (3)

Move away from mpn_addmul_1!

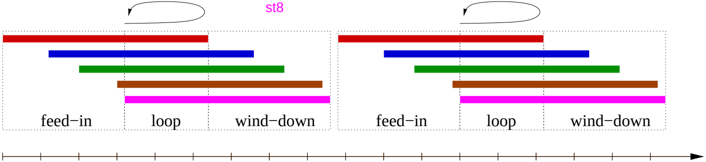
Assembly strategies (3)

Move away from mpn_addmul_1!

- Make mpn_addmul_2 the main primitive
- Or mpn_addmul_3, etc.
- Base mpn_mul_basecase on these
- Base a mpn_redc_N for each mpn_addmul_N

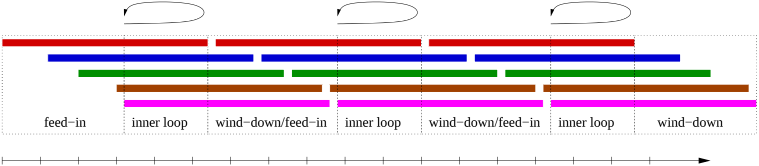
OSP: Simple Software Pipelining

ldf8 ldf8
xma.l xma.hu
getf.sig getf.sig
add cmp.ltu add cmp.eq.or
st8

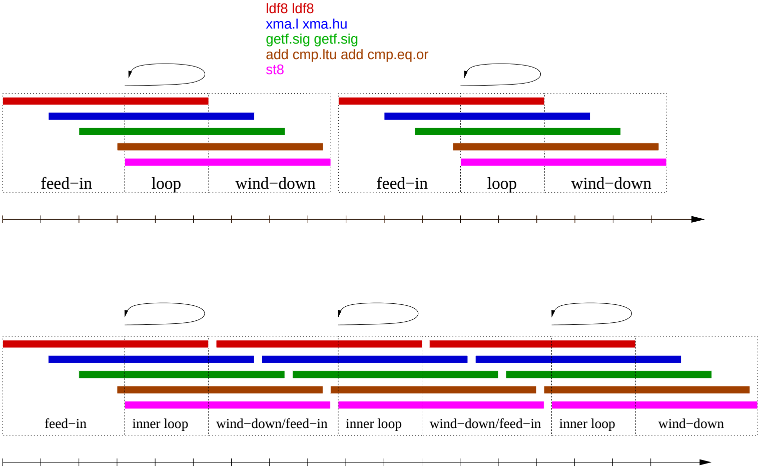


Overlapped Software Pipelining (OSP)

ldf8 ldf8
xma.l xma.hu
getf.sig getf.sig
add cmp.ltu add cmp.eq.or
st8



OSP: Simple s/w Pipelining vs OSP



ldf8 ldf8
xma.l xma.hu
getf.sig getf.sig
add cmp.ltu add cmp.eq.or
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OpenSSL's and assembly

- Inadequate primitives
- Assembly is fast, period
- More is better

The answer to the initial question: NO.

GMP 4.2 division

	Left-to-right	Right-to-left/REDC
Schoolbook on addmul_1	Y	Y
Schoolbook on addmul_N	–	Y
Burnikel-Ziegler $O(M(n)\log n)$	–	Y
Barrett-MUA $O(M(n))$	–	–

GMP 5 division

	Left-to-right	Right-to-left/REDC
Schoolbook on addmul_1 + OSP	Y	Y
Schoolbook on addmul_N + OSP	Y	Y
Burnikel-Ziegler $O(M(n) \log n)$	Y	Y
Barrett-MUA $O(M(n))$	Y	Y

Burnikel-Ziegler division

```
mp_limb_t
mpn_dc_bdiv_qr (mp_ptr qp, mp_ptr np, mp_srcptr dp, mp_size_t n, mp_limb_t dinv
{ lo = n >> 1; /* floor(n/2) */
  hi = n - lo; /* ceil(n/2) */

  if (BELOW_THRESHOLD (lo, DC_BDIV_QR_THRESHOLD))
    cy = mpn_sb_bdiv_qr (qp, np, 2 * lo, dp, lo, dinv);
  else
    cy = mpn_dc_bdiv_qr_n (qp, np, dp, lo, dinv, tp);

  mpn_mul (tp, dp + lo, hi, qp, lo);

  mpn_incr_u (tp + lo, cy);
  rh = mpn_sub (np + lo, np + lo, n + hi, tp, n);

  if (BELOW_THRESHOLD (hi, DC_BDIV_QR_THRESHOLD))
    cy = mpn_sb_bdiv_qr (qp + lo, np + lo, 2 * hi, dp, hi, dinv);
  else
    cy = mpn_dc_bdiv_qr_n (qp + lo, np + lo, dp, hi, dinv, tp);

  mpn_mul (tp, qp + lo, hi, dp + hi, lo);

  mpn_incr_u (tp + hi, cy);
  return rh + mpn_sub_n (np + n, np + n, tp, n);
}
```

Relevance to cryptography

- All RSA sizes benefit from
  - mpn_addmul_2 (mpn_addmul_3, . . .) in multiplication
  - mpn_addmul_2 (mpn_addmul_3, . . .) in REDC
  - OSP in multiplication
  - OSP in REDC
- Larger RSA sizes (2048-) benefit from
  - Burnikel-Ziegler REDC

64-bit vs 32-bit cryptography

	512	1024	2048	4096
Athlon32	566950	3051800	19415000	130360000
Athlon64	170370	759670	4690800	30137000
A32/A64	3.33	4.02	4.14	4.33

Athlon32:
MUL_KARATSUBA_THRESHOLD 26 (832 bits)
SQR_KARATSUBA_THRESHOLD 52 (1664 bits)

Athlon64:
MUL_KARATSUBA_THRESHOLD 25 (1600 bits)
SQR_KARATSUBA_THRESHOLD 80 (5120 bits)